

MINIMUM COST STEINER TREE PROBLEM

Gustavo Bergantiños¹, Leticia Lorenzo² y Silvia Lorenzo-Freire³

¹ Research Group in Economic Analysis. Universidade de Vigo.

² Research Group in Economic Analysis. Universidade de Vigo.

³ Universidade da Coruña

ABSTRACT

Consider a group of agents located in different geographical places that are interested in some resource that can only be provided by a common supplier. Agents can be served directly from the supplier or indirectly through other agents or some public nodes, also called switches. The connection between two agents, an agent and the source, an agent and a public node, or a public node and the source entail some cost. The first problem that we need to solve is how to provide all the agents with the resource with a minimal total cost. This problem is known as the Steiner tree problem and it has been proved to be NP-hard (Gilbert and Pollack, 1968; Garey et al, 1977). So in order to find a good solution, heuristics need to be applied. Assume that we know the minimal Steiner tree and the costs associated with all the connections among the agents, the source, and the different public nodes. Given this situation the question that arises is how to allocate the cost of constructing this minimal Steiner tree among the agents? Meggido (1978) shows that the problem of allocating the cost among the agents may have an empty core. This means that in some cases is impossible to obtain an stable allocation where no group of agents prefer to construct and pay their own network.

Thus, we define a new class of problems, we call them minimum cost Steiner tree problems. A minimum cost Steiner tree problem is a classical minimum cost spanning tree problem where we also have information about an approximated Steiner tree, obtained with some software, that connects all the agents to the source, using some public nodes, and with a cost strictly less than the cost of constructing a minimal tree without using the public nodes. Given a minimum cost Steiner tree problem, since we know the tree that is going to be constructed in order to provide all the agents with the resource, the problem that arises next is how to allocate the cost of constructing that tree among the agents. We associate a cooperative TU game with this situation and study the core.

Keywords: OR games, minimum cost spanning tree problems, Steiner trees, core selection.