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MODELLING UNCERTAINTY IN DECISION MAKING: FUZZY PERFORMANCE MEASURES AND FUZZY TARGET

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ABSTRACT

Decision making processes are based on performance measures that are compared with other performance measures values or with targets. Generally, the performance measure is only a number which do not consider the uncertainty associated with obtaining it such as measurement errors, precision and accuracy of measurement tools and subjective judgment. In this work, the uncertainty associated with performance measures is quantified through fuzzy numbers. The decision criterion is also represented by a fuzzy number. An indicator that considers both uncertainties in performance measure and decision criterion is developed. This indicator will help the decision maker once it quantifies the risk associated with the decision.

Keywords: uncertainty, fuzzy targets, decision making, performance measures.

1. INTRODUCTION

The uncertainty is a quantitative indication of the quality of the result. It is an unavoidable part of any measurement and it starts to matter when results are close to a specified limit. When uncertainty is evaluated and reported in a specified way it indicates the level of confidence that the value actually lies within the range defined by the uncertainty interval.

Uncertainty is a consequence of unknown random and systematic effects and is therefore expressed as a quantity, i.e., an interval about the result. “When reporting the result of a measurement of a physical quantity, it is obligatory that some quantitative indication of the quality of the result be given so that those who use it can assess its reliability. Without such an indication, measurement results cannot be compared, either among themselves or with reference values given in a specification or standard. It is therefore necessary that there be a readily implemented, easily understood, and generally accepted procedure for characterizing the quality of a result of a measurement, that is, for evaluating and expressing its uncertainty.” (JCGM/WG 1, 2008, p. viii). This is common knowledge in metrology but it is not being applied in ordinary Performance Measures (PMs). Thus the quality of a result can be expressed through the uncertainty associated with such PMs.

There is a wide variety of reasons why uncertainty is present in Performance Management Systems (PMSs). Particularly, to reliability studies Coolen (2004) presents three main reasons: (i) in many reliability applications, there may be few, if any, statistical data available, implying stronger dependence on subjective information in the form of expert judgments; (ii) the relaxation of dependence on precise statistical models justified by physical arguments; (iii) an assumption underlying most mathematical work in the study of system reliability is that the exact system structure and dependence relations between components are known, which may well be unrealistic in many applications for all but the simplest systems. These relationships are conditioned by the system’s environment and may generate contradictory information, vagueness, ambiguity data, randomness, etc. In reliability studies, the vagueness of the data have many different sources: it might be caused by subjective and imprecise perceptions of failures by a user, by imprecise records of reliability data, by imprecise records of the tools appropriate for modelling vague data, and suitable statistical methodology to handle these data as well (Nunes et al., 2006).

Both Wazed et al. (2009) and Mula et al. (2006) considered uncertainty in manufacturing systems and argue that reducing it, is a means to improve the system. Other studies have included uncertainty in project scheduling (Herroelen and Leus, 2005), inventory control (Petrovic et al., 1999), or supply chain management (Petrovic, 2001).

In the traditional formulation of a PMS, most PMs are affected by imprecision and vagueness but they are represented using numerical crisp values. A good decision-making model needs to tolerate vagueness and imprecision because these types of the non-probabilistic uncertainty are common in decision-making problems (Yu, 2002).

Fuzzy Set Theory and Fuzzy Logic have proved to be a successful in handling imprecise and vague knowledge that characterize this kind of problems, and it has been applied in a variety of fields in the last decades.

If the uncertainty (fuzziness) of human decision-making is not taken into account, the results can be misleading (Lee et al., 2008).

Generally, each PM is represented by a number that is not able to represent uncertainty. The problem is how to overcome this situation or how to deal with data uncertainty. Several ways can be used to represent the uncertainty attribute evaluation (Durbach and Stewart, 2011) such as: standard deviation, probability distribution, expected values, fuzzy numbers, scenarios and quartiles.

In this work, the uncertainty in PMs and decision criteria are represented through fuzzy set in order to create a risk indicator associated with the decision.

2. MODELLING FUZZY PERFORMANCE MEASURES AND FUZZY TARGETS

The numerical assessment of fuzzy parameter/data and linguistic variables such as some PMs on customer satisfaction is done by using adequate membership function which determines the degree of membership in each input fuzzy set. The design of a fuzzy model is not trivial and several approaches (Ross, 1995) (Klir et al., 1997) have been proposed to identify the shape of elementary PMs. However, this subject will not be discussed in this work and the most usual solution is to use triangular and/or trapezoidal membership functions. The selection of these functions (see examples in Figure 1) has advantages in terms of their manipulation.

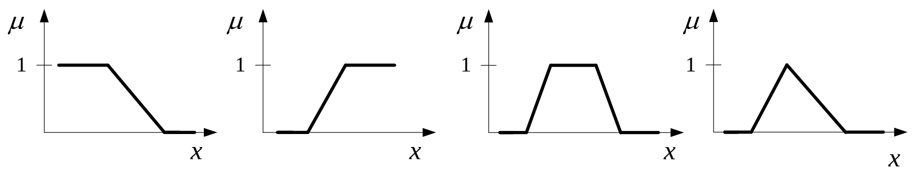


Figure 1: Commonly used membership functions

In order to ensure that companies achieve their goals and objectives, PMs are used to evaluate, control and improve processes. PMs are also used to compare the performance of different organizations, plants, departments, teams and individuals and to assess employees (Donald and Chan, 2002). Green et al (1991) suggest the introduction of PMs that relate to the needs and goals of the organisation. Obviously, if models incorporate the uncertainty associated to some parameters (e.g. by using fuzzy numbers) to estimate some components, than the result is itself fuzzy. Let $M = f(x_1, x_2, x_3, \ldots, x_n)$ be the analytical model for a given crisp PM *X*. This model maps the *n* inputs $(x_1, x_2, x_3, \ldots, x_n)$ into the output space *X*. It is now intended to extend such a mapping to fuzzy sets $\tilde{M} = f(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \ldots, \tilde{x}_n)$.

Uncertainty propagation

The arithmetic by intervals and the Zadeh's extension principle (Zadeh, 1978) constitute the two fundamental methods to handle fuzzy numbers. These methods have shown to be suitable for simple mathematical operations, but if the analytical methods are complex, these may lead to fuzzy results exhibiting a spreading larger than the required minimal spreading as is showed by Nunes (2005).

The membership function of the PM, $\tilde{M}$ is a surface with the possible values of M. Under these circumstances arises the difficulty of interpreting the result. Frequently, a defuzzification is performed of the membership function of the PM to obtain a crisp number. In this operation a lot of information is lost that could be relevant to the decision process. Thus the fuzzy result is richer than the crisp number.

Let $\tilde{Z}$ be the membership function of the acceptable performance level or fuzzy target for the PM $\tilde{M}$. This function represents the acceptance/rejection region of the decision maker. Frequently the decision maker does not have a clear defined boundary between acceptable and unacceptable results. $\tilde{Z}$ can be represented by a fuzzy membership function:

$$\tilde{Z}(x) = \begin{cases} 0 \rightarrow x \leq c_1 \\ \psi(x) \rightarrow x \in [c_1, c_2] \\ 1 \rightarrow x \geq c_2 \end{cases}$$

where,

$\tilde{Z}$ is the membership function of the acceptable performance level

$\psi(x)$ is the functional relation representing the subjective opinion of the acceptable risk

c_1 is the inferior limit of the acceptable region

c_2 is the superior limit of the not acceptable region

The ability to let the decision maker to choose the limits c_1 and c_2 and the function $\psi(x)$ allows the introduction of ambiguity on the risk acceptance of different decision makers. One value of M between on the region of doubtful acceptance is considered an Acceptable Value with a membership level α, with 0 < α < 1. If that value is in the acceptable/unacceptable region its value would be α=1 or α=0, respectively. For example, in Figure 2, both the PM M and the target Z are fuzzy numbers expressed $\tilde{M}$ and $\tilde{Z}$.

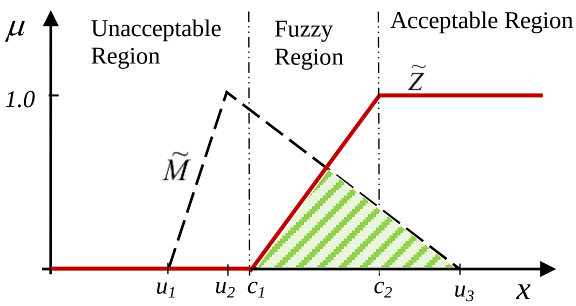


Figure 2: Fuzzy PM and fuzzy target

3. COMPLIANCE INDICATOR

In Nunes and Sousa (2009), the *Compliance* of two membership functions $\tilde{M}$ and $\tilde{Z}$ was calculated as a fuzzy measure of compatibility. There are several measures to quantify the compatibility of two fuzzy numbers (El-Baroudy and Simonovic, 2003). The overlapping area between two membership functions (i.e. a fraction of the total area of the PM) represents the concept of *compliance* better than other compatibility measures such as possibility and necessity. Thus,

$$Compliance = \frac{\text{overlaping area of membership functions of PM and target}}{\text{total area of membership funtion of PM}} \qquad (1)$$

The following assumptions are considered when using *Compliance* (C):

- The maximum of C is equal to 1, and that happens for any level of α such as, $\tilde{X}(\alpha) \geq Z \quad \forall \alpha \in [0,1]$
- The minimum value of C is equal to 0, and that happens for any level of α, such as $\tilde{X}(\alpha) < Z \quad \forall \alpha \in [0,1]$

– The C provides a consistent ranking to assess the degree to which a fuzzy number complies with target. It is a monotonic increasing function.

Considering the situation represented in Figure 3 where $\tilde{M}$ and $\tilde{M}'$ are two fuzzy PMs with similar membership functions concerning shape, but with different universe of discourse. The universe of discourse of $\tilde{M}$ is divided between the acceptable region (50%) and the fuzzy region (50%) while the universe of discourse of $\tilde{M}'$ is totally in the acceptable region.

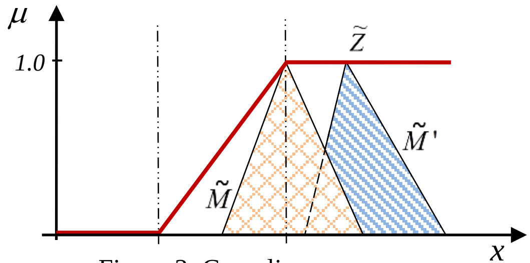


Figure 3: Compliance cases

Using the compliance measure given by equation (1) for both situations $\tilde{M}$ with $\tilde{Z}$ and $\tilde{M}'$ with $\tilde{Z}$, results are identical. However, for a decision Maker, the two situations are completely different!

Moreover, for the situation described by Figure 4, $\tilde{M}$ e $\tilde{M}'$ have identical areas and both have the same area below the membership function $\tilde{Z}$ and the low significant area and the high significant area indicated in the figure are identical. However, an overlap in high significance area (closer to the acceptance region) is preferable than an overlap in low significance area.

For both situations, the compliance measure obtained by equation (1) gives also identical values for $\tilde{M}$ with $\tilde{Z}$ and $\tilde{M}'$ with $\tilde{Z}$, despite having different degrees of agreement with the target $\tilde{Z}$ Therefore, the compliance measure should take into account the weighted area approach.

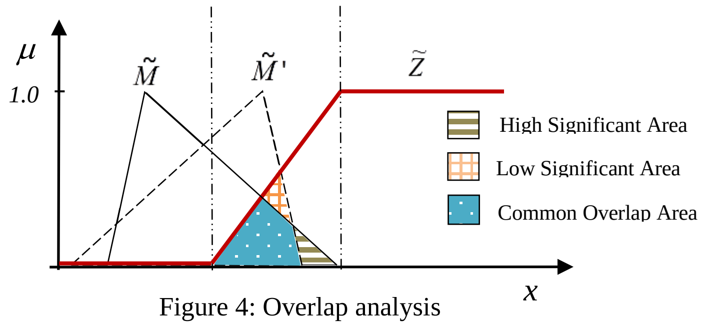


Figure 4: Overlap analysis

A new indicator is proposed in equation (2). This indicator also considers the area below the membership function of the target ($\tilde{Z}$) for the numerator but assigns a weigh based on the membership value ($\mu_{\tilde{Z}}$ (x)) of the decision criterion. The expression in the numerator determines the universe of discourse. The extreme values used in the integral, u_1 and u_3, represent the extreme values of the universe of discourse of the membership function of PM ($\tilde{M}$) as shown in Figure 2.

When the (fuzzy) PM is possible in the fuzzy region of the decision criterion (i.e. the membership function of the PM has values greater than zero in the interval where the membership function of the fuzzy target has values between 0 and 1), each possible value of the PM is weighted by the membership function of the fuzzy target. For example, if the values of the PM are highly possible to fulfil the decision criterion, the bigger will be its weight, and smaller will be the risk of not achieving the target.

$$C = \frac{\int_{u_1}^{u_3} \mu_{\tilde{M}}(x) \cdot \mu_{\tilde{Z}}(x)\, dx}{\int_{u_1}^{u_3} \mu_{\tilde{M}}(x)\, dx} \qquad (2)$$

Considering the triangular membership function for PM, M (represented in Figure 2), its value will be given by:

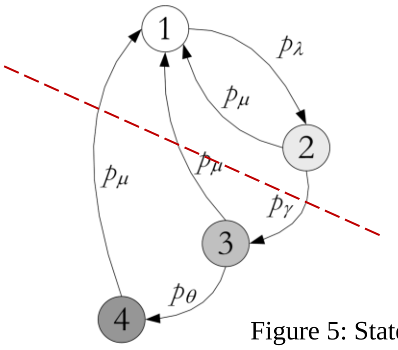
$$\mu_{\tilde{M}}(x) = \begin{cases} \dfrac{x-u_1}{u_2-u_1} & u_1 < x \le u_2 \\ \dfrac{u_3-x}{u_3-u_2} & u_2 < x \le u_3 \end{cases} \tag{3}$$

The membership function of the target, also represented in Figure 2, is given by:

$$\mu_{\tilde{Z}}(x) = \begin{cases} 0 & x \le c_1 \\ \dfrac{x-c_1}{c_2-c_1} & c_1 < x \le c_2 \\ 1 & x > c_2 \end{cases} \tag{4}$$

4. NUMERIC ILUSTRATIVE EXAMPLE

To illustrate the model, a practical example will be based on one single PM "*system availability*" and the decision is to consider it *available* or *not available (as described by Figure 7 (red line))*. Figure 5 represents the behaviour of a system composed by a main repairable component, and a stand by component. This later component, once activated, is able to ensure the normal operation of the system just for a limited period of time. State 1 corresponds to the normal operation of the system. When the main component fails (process p_λ), the system enters state 2 where a reconfiguration process (process p_γ) becomes active in order to put into operation the stand-by component. Once activated (state 3), this component is able to operate only for a limited period of time due to an overheating (process p_θ). If that time elapses before the accomplishment of the repair operation (process p_μ), a global system failure will occur (state 4).



Process	Rate of transition
p_λ	λ
p_γ	γ
p_θ	θ
p_μ	μ

Figure 5: State Diagram

The fuzzy model parameters used in this study are analytically represented by the membership functions expressed by:

$$\tilde{\lambda}(x) = \begin{cases} 0 & x < 0.02 \ and \ \ x \ge 0{,}04 \\ 1 & x = 0.02 \\ (0.04-x)/0.02 & 0.02 \le x < 0.04 \end{cases} \qquad \tilde{\gamma}(x) = \begin{cases} 0 & x < 2/3 \ \ and \ \ x \ge 2 \\ (x-2/3)/1/3 & 2/3 < x < 1 \\ (2-x) & 1 \le x < 2 \end{cases}$$

$$\tilde{\theta}(x) = \begin{cases} 0 & \mathrm{x} < 1 \ \text{and} \ \ \mathrm{x} \ge 2 \\ 1 & \mathrm{x} = 1 \\ (2-x) & 1 \le \mathrm{x} < 2 \end{cases} \qquad \tilde{\mu}(x) = \begin{cases} 0 & \mathrm{x} < 1/5 \ \text{and} \ \ \mathrm{x} \ge 1/3 \\ (x-1/5)/(1/20) & 1/5 < \mathrm{x} < 1/4 \\ (1/3-x)/(1/12) & 1/4 \le \mathrm{x} < 1/3 \end{cases}$$

When all the processes that govern the behaviour of a system have, or are assumed to have, exponential distributions, the theory of Markov processes provides a powerful foundation for its evaluation. The analysis of a Markov process allows the computation of state probabilities $P_i(t) = P\,[Z(t) = i]$ either for transient or steady-state behaviour, based on the Komogorov differential equation. For steady-state the state probabilities can be obtained from the following set of equations:

$$\begin{cases} P \times Q = 0 \\ P \times e = 1 \end{cases} \quad (5)$$

Where P=[P_1, P_2,…, P_n] is the state probability vector, Q is a square matrix of dimension n with elements q_{ij} given by the rate of transition from state i to state j ($j \neq i$) and elements q_{ii} given by the rate at which the system departs from state i, i.e., $q_{ii} = -\sum_{j, j\neq i} q_{ij}$, and e=[1, 1, 1, …,1]^T results from the fact that the sum of the states' always probabilities equals to one.

The analytical expressions of the states' probabilities in the steady state, Pi, obtained by the Markov Chains method, through the resolution of the system of equations (5) are the following:

$$P_1 = \frac{\mu}{\mu + \lambda} \qquad P_2 = \frac{\lambda\mu}{\gamma\lambda + \gamma\mu + \lambda\mu + \mu^2} \qquad P_3 = \frac{\lambda\mu\gamma}{(\gamma+\mu)(\lambda+\mu)(\mu+\theta)} \qquad P_4 = \frac{\lambda\theta\gamma}{(\gamma+\mu)(\lambda+\mu)(\mu+\theta)}$$

Using the analytical expressions of the state probabilities and the membership functions of the input variables represented above, the membership functions of the states probabilities can be obtained by making a proper propagation of uncertainty of inputs through the analytical models of the states probabilities.
It should be noted that the simple use of the arithmetic mean interval does not allow in this case an appropriate propagation of uncertainty because the analytical expressions of the states probabilities are a function of diffuse parameters appearing at the same time as numerator and denominator (Nunes, 2005).

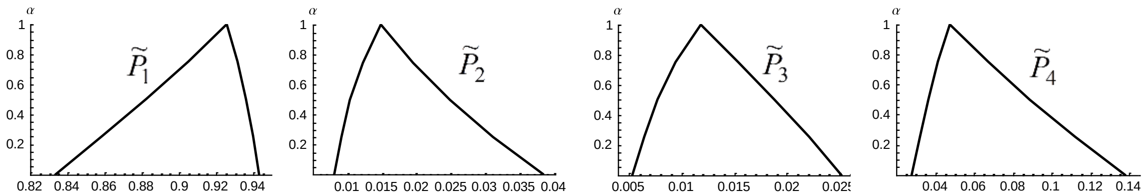


Figure 6: Membership functions of state probabilities

Knowing the states probabilities of the system in the steady state one can calculate the system availability (the probability of the system play its mission) by the sum of the probabilities of its operational states (states where the system fulfils the mission). In this example it is assumed that the system fully fulfil its mission only in states 1 and 2. In these circumstances the availability membership function, obtained by $\widetilde{A} = \widetilde{P}_1 + \widetilde{P}_2$ presents a triangular membership function represented in Figure 7. This figure represents also the Target Z function for PM A. Thus, for this case, we have: u_1=0.84142; u_2=0.940741; u_3=0.981858; c_1=0.8; c_2=0.94

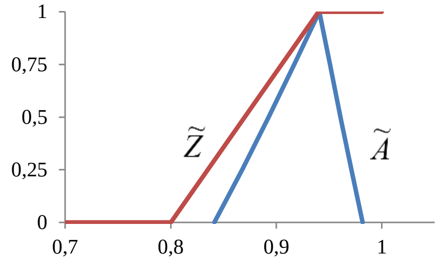


Figure 7: Membership function of Availability

By using the equation (2) with these parameters, the value of 0,8336 is obtained for the indicator C. This indicator gives us a measure of agreement between PM A and the fulfilment of the target Z. Otherwise, the difference (1-C) represents the "distance" of PM A from the fulfilment of target Z.

5. CONCLUSIONS

The representation of uncertainty in performance measurement systems is at the centre of this on-going research. This part of the research presents the findings of this deductive research which will later be tested through case studies, to allow another step of inductive research to support, change or refute the proposed elements for the PMS.

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