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AN OVERVIEW ON RECENT APPROACHES FOR VEHICLE ROUTING PROBLEMS WITH LOADING CONSTRAINTS

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ABSTRACT

Most of the approaches developed for vehicle routing problems require only that the total weight or volume of the load does not exceed the capacity of the vehicles. For many real distribution applications, these approaches are not adequate, since they lead most of the time to suboptimal or even infeasible solutions. Here, we focus on those applications where the loading of the vehicles is a difficult issue that must be considered explicitly when planning the routes.

The vehicle routing problem with loading constraints is NP-hard since it combines two well-known and difficult problems, namely the vehicle routing problem with the packing problem. The study of these problems has motivated an increasing interest and approaches in recent times. The vast majority of the approaches described in the literature focus on heuristic approaches. They address different variants of the problem. Developing exact approaches and strong models for these problems remains a true challenge. In this paper, we provide an updated survey on recent contributions on that field.

Keywords: Vehicle Routing Problem, Loading constraints, time windows.

1. INTRODUCTION

The Vehicle Routing Problem (VRP) is one of the most well-known and studied combinatorial optimization problems. To the best of our knowledge, the first work related to VRP was published in 1959 (Dantzig and Ramser, 1959). Since then, many contributions appeared in the literature, resulting in increased closeness between the proposed methods and the real context in which these problems arise. In a generic way, the VRP consists in determining a set of optimal routes to be covered by a fleet of vehicles to serve a set of customers while minimizing the involved costs. In this general problem, there can be additional constraints. The most basic are related to the vehicle capacity. The corresponding problems are known as Capacitated Vehicle Routing

Problems (CVRP). Other restrictions and characteristics have been considered such as time intervals in which the deliveries have to be processed, clients that may be served only once and by a single vehicle, or vehicles having different capacities. An exhaustive review of these problems can be found in (Toth and Vigo, 2002).

The most usual constraints treated in the literature have been with no doubt related to the capacities of the vehicles. However, in most of the cases, the approaches that are proposed assume that the capacity is a simple one-dimensional measure (typically a maximum weight or volume). In many real applications, this approach may not be adequate. That happens for example when the vehicles travel with their maximum load. In these cases, distributing the loads among the available space may be a real issue because of the loading constraints that are involved. These problems are described in the literature as CVRP with loading constraints (L-CVRP). The L-CVRP integrates two hard combinatorial problems, resulting in a very challenging optimization problem. Indeed, the methods applied to this variant have to consider both the definition of the routes and the packing problem (BPP) in either two- (2L-CVRP) or three-dimensions (3L-CVRP).

Another important issue that must be taken into account in L-CVRP is the relative position of the items in the vehicles, and its relation with the sequence of visits that the vehicle has to do. In practice, it is important that the delivery of an item to a costumer does not require moving the other items that are in the vehicle. Therefore, the items must be placed inside the vehicle according to the order by which the customers will be visited. The problems that consider explicitly this constraint are known as sequential L-CVRP. For the L-CVRP that do not include this constraint, the designation unrestricted L-CVRP is used instead.

The items may have a fixed or a variable orientation. Usually, these constraints are related to the way the vehicles are loaded. Indeed, they can be either rear-loaded, side-loaded or loaded in both ways. Sometimes, in 3L-CVRP, it may happen that the items cannot be placed upside-down, or over each other. In this latter case, the problem reduces to a 2L-CVRP.

Due to its inherent complexity, the majority of the solution methods proposed for the L-CVRP consist in heuristics. In fact, there is only one reference in the literature describing an exact solution method (Iori et al., 2007). In this paper, we provide an updated review of the variants of the L-CVRP studied in the literature and the corresponding approaches proposed for their resolution.

2. 2L-CVRP

As referred to above, heuristics are the most common solution methods that are used to solve the L-CVRP. In (Gendreau et al., 2008), the authors proposed a tabu search method for the 2L-CVRP. The movements that characterize their neighbourhood structure consist in exchanging a customer from a route to another. After they have been applied, these moves are inserted into their tabu list. Infeasible routes are allowed in the sense that the weight or the height of the loading area can be exceeded. In these cases, the movements are penalized according to the dimension of this violation. The results reported by the authors show that this approach is competitive with exact approaches, finding the optimal solution in roughly 50% of instances presented in (Iori et al., 2003). Gendreau *et al.* conclude that the loading constraints have an important impact on the costs of the solutions.

Zachariadis *et al.* (Zachariadis et al., 2009) presented a guided tabu search algorithm for the 2L-CVRP. In their approach, a heuristic bundle is adopted to determine the feasibility of a route from the loading point of view. In order to accelerate the convergence of the algorithm, they adopted two procedures: the neighbourhood is reduced in each iteration, and information relative to the feasibility of the routes is stored. Thus, it is not necessary to re-evaluate whether the solution is feasible or not. The results of this approach point out the adequacy of this method, since it outperforms the results of a large number of instances of the literature.

The first work reporting on the 2L-CVRP whose items may have a variable orientation is due to Fuellerer *et al.* (Fuellerer et al., 2009). In this case, items are allowed to be rotated 90 degrees in the horizontal plane. The authors describe an Ant Colony Optimization algorithm (ACO), which is an extension of the ACO for the CVRP. The ACO algorithm tries to compute the vehicle routes. It is necessary to check the feasibility of these routes in terms of the loading constraints. For this reason, the authors resort to lower bounds based on the procedure described in (Martello and Vigo, 1998) but extended to the case in which items have a variable orientation. If it is not possible to prove the infeasibility of a given route using these lower bounds, the items are sorted, and then two heuristics are applied: a bottom-left filling heuristic and a *Max Touching Perimeter Algorithm*. If both these heuristics fail, a local search algorithm is applied, switching items in the sequence of input in the referred heuristics, and executing again other the two heuristics. If this alternative fails, a truncated branch-and-bound method is then applied. The authors report on results which improve by 5% and 2,5% the results achieved by (Gendreau et al., 2008) and (Zachariadis et al., 2009), respectively.

Based on the work of (Zachariadis et al., 2009), Leung *et al.* (Leung et al., 2011) presented an extended guided tabu search algorithm for the 2L-CVRP. In this extended version, a movement is accepted if it produces a new incumbent solution, even if it is in the tabu list. The authors also suggest a new heuristic procedure to be added to the heuristic bundle referred above, namely the lowest reference line best-fit heuristic. In their paper, they report on small improvements compared to those in (Zachariadis et al., 2009) for the case where their heuristic procedure is not used. The improvements become more significant when this heuristic is included in the heuristic bundle.

More recently (2011), a multi-start evolutionary local search algorithm was presented with quite good results (Duhamel et al., 2011). This method combines a greedy randomized adaptive search procedure with an evolutionary local search method. One of the innovative feature of this approach is the handling of loading constraints: these constraints are incorporated in a resource constrained project scheduling problem (RCPSP). At the end of the process, the solution of this RCPSP is converted back into a solution for the 2L-CVRP. The computational tests were performed on the same instances presented in (Gendreau et al., 2008; Zachariadis et al., 2009; Fuellerer et al., 2009). The authors report on computational results that outperform the other approaches developed so far.

Developing an exact procedure to the 2L-CVRP is clearly a challenge given the complexity of the involved problems. To the best of our knowledge, the only exact approach proposed in the literature dates back to 2007 and it is due to Iori *et al.* (Iori et al., 2007). Their method is based on a branch-and-cut algorithm that is used to find the optimal integer solution of a two-index vehicle flow model with capacity-cut constraints and path constraints. The former guarantees that a solution does not violate the weight and loading constraints nor it allows overlap between items. The later prevents solutions in which sequential loading is not possible. The capacity-cut constraints depend on the minimum number of vehicles. In each iteration, a branch-and-bound algorithm verifies if the loading is feasible. Additionally, heuristics are used to search for violated inequalities. This approach presents good results for small size instances. For larger instances, the quality of the results decreases naturally.

3. 3L-CVRP

The first work referring to the combination of CVRP with three-dimensional loading constraints was proposed in 2006 by Gendreau *et al.* (Gendreau et al., 2006). These authors proposed a tabu search algorithm that allows movements that produce infeasible solutions, generalizing the algorithm proposed for the 2L-CVRP (Gendreau et al., 2008). The loading of the vehicles is achieved through the application of an inner tabu search algorithm which explores the neighbourhood by changing the loading sequence of the items. From this new sequence, two heuristic procedures are applied. These heuristics are extended versions of the bottom-left and the touching perimeter heuristic.

Later, Fuellerer *et al.* (Fuellerer et al., 2010) proposed a modified ant colony optimization for the same problem and tested it using the same set of instances as those used in (Gendreau et al., 2006). In (Iori and Martello, 2010), these methods were compared with the guided tabu search proposed in (Zachariadis et al., 2009). Their results show that the ant colony optimization yields the best results with an improvement of 8% and 4% compared to tabu search and to the guided tabu search approach, respectively.

Some works consisting in extensions of other approaches for the 2L-CVRP are described in (Tarantilis et al., 2009; Duhamel et al., 2011).

Bortfeldt (Bortfeldt, 2010) proposed an original approach consisting in a tabu search procedure for defining the routes and and in a tree search algorithm for the loading part of the problem. The tabu search algorithm is initialized randomly according to the multi-start randomized savings procedure described in (Toth and Vigo, 2002). The best movement is chosen if it is not in the tabu list and if it is feasible. The inverse movement is inserted in the tabu list. The author reports on results that outperform the ones obtained in (Gendreau et al., 2006; Tarantilis et al., 2009; Fuellerer et al., 2010).

In (Aprile et al., 2007), Aprile *et al.* addressed a variant of the 3L-CRVP arising in the production of sofas. The authors proposed to pre-compute the loading by packing the items of each costumer separately. After matching pairs of customer orders, a simulated annealing approach is applied. The proposed algorithm was able to solve instances with up to 100 customers in less than 90 seconds.

4. 3L-CVRP WITH TIME WINDOWS

Some authors addressed another variant of these problems in which the items have to be delivered to the customers within a given time window. Moura and Oliveira (Moura and Oliveira, 2009) developed two approaches for the 3L-CVRP with time windows: a sequential method and a hierarchical method. The sequential method consists in building a sequential candidate list (SCL) which will be used by three distinct search methods. The first one is a Monte Carlo procedure which generates random solutions from the SCL. Another approach is based on the Local Search heuristic through the 2-opt strategy whose neighbourhood will be composed by the SCL. Finally, the sequential approach is also composed by a GRASP procedure whose elite list is built according to a given ranking. The local search phase of GRASP is based on a 2-opt strategy. The hierarchical approach is composed by three phases: constructive, post-constructive and local search. In the constructive phase, a GRASP algorithm is used only to build the routes. Then, a post-constructive phase tries to minimize the number of routes. In the local search phase, a 2-opt procedure is applied on the routes and between the routes. The authors used new instances to test their approach by considering the variation of customers demand. Through the analysis of the results, the authors conclude that for a smaller number of boxes, sequential and hierarchical approaches are similar. However, if the number of boxes is higher, the GRASP algorithm (in the sequential approach) leads to better solutions.

Moura (Moura, 2008) proposed a multi-objective genetic algorithm for the same problem. The initial population is obtained through the application of the sequential approach referred above (Moura and Oliveira, 2009). The chromosome is represented by a string of equal length of the sequential candidate list. Each gene of a given chromosome represents a candidate of this list. To evaluate a given solution, only two objectives (number of vehicles and distance) are considered, through a Pareto ranking procedure, which will be used to build a potential non-nominated solutions. Two solutions will be randomly chosen to recombination. The author report on competitive results compared with (Moura and Oliveira, 2009) using the same instances.

5. CONCLUSIONS

In this work, we review some of the most recent approaches proposed for the L-CVRP. Both exact and heuristics methods were discussed. As mentioned above, heuristics are the most common approaches. The exact algorithm proposed in the literature may be adequate for small instances, but for larger (real) instances, heuristics remains the only effective approach. In fact, developing exact or even hybrid methods for the L-CVRP remain an unexplored area that is clearly a challenge.

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