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On the effect of taxation in the online sports betting market[1]

Juan Vidal-Puga[1]

[1]Universidade de Vigo

SUMMARY

We analyze the effect of taxation on the online sport betting market. This market is characterized by its negligible marginal costs. Taxation can be on volume (General Betting Duty) or on gross profit (Gross Profit Tax). We model the two most popular online sport betting bets: fixed-odds and spread, as compared with other traditional sport betting: parimutuel. We show that from the bettor's point of view taxation on gross profit is always preferable to taxation on volume.

Keywords: taxation, online betting market.

1. INTRODUCTION

Over the last few years, many European countries have been regulating their online betting and gaming sector. However, this regulation has not been done in a uniform way throughout the different countries.

In general, the basic taxation scheme is based on two types of taxes: the General Betting Duty (GBD) is levied as a proportion of betting stakes; whereas the Gross Profits Tax (GPT) is levied as a proportion of the net revenue of the operators.

Some examples: the United Kingdom applied a 6.75% tax on GBD until October 2001, when it was replaced by a 15% tax in GPT (National Audit Office (2005)). Italy applies a 2%-5% tax on GBD (Ficom Leisure Advisory & Investments (2011)) since March 2011. France applies a 8.5% tax on GBD (Global Betting & Gaming Consultants (2011)) since 2010. Cyprus applied a 10% tax on GPT until March 2011, when it was replaced by a 3% tax on GBD[2]. Recently, Spanish authorities approved a law that applies a 25% tax on GPT for some types of bets and a 15-22% tax on GBD for others, plus a general 0,1% tax on GBD (BOE 127(I), 52976-53022). In Table 1 we summarize the data.

In the Spanish case, GBD has been the taxation scheme in the most traditional offline sport betting (*la quiniela*), which takes a parimutuel structure.

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[2]http://online.casinocity.com/jurisdictions/cyprus/

Table 1: Taxation in some European countries.

	GBS	GPT
UK	6.75% (until 2001)	15% (since 2001)
Italy	2-5%	-
France	8.5%	-
Cyprus	3% (since 2011)	10% (until 2011)
Spain	25% (parimutuel)+0.1%(general)	15-22% (fixe odds and spread)

In a parimutuel market, a winning bet pays off a proportional share of the total stake on all outcomes. However, the most popular online sport operators are specialized in another two markets: Fixed-odds and spread. In a fixed-odd market, the operator sets the odds for each possible outcome of the match, and the bettors decide whether they accept or not these odds. In a spread market, the operator acts as a intermediator among the users, whom bargain the odds.

For sport matches, a bet of 1 monetary unit on a particular team yields a return of $\frac{1}{\pi}$ monetary units in case the team wins the match, and 0 otherwise. In this context, an odd $\pi \in (0,1)$ is defined as the probability assigned by the market. Notice that any risk-neutral bettor would find it profitable to bet at odd π when its private probability estimation is higher than π.

In parimutuel and spread bets, the operator's profit comes from a commission on either the amount at risk or the winning amount (typically a 5% in online spread operators). In fixed-odds bets, the operator's profit comes from the odds, which should sum up more than 100%[3] for all the possible outcomes of the sport match[4].

In this paper, we model the three types of market in a general setting. The regulator decides on the general taxing scheme (either GBD or GPT) and the operators decide on their commission (parimutuel and spread operators) or odds (fixed-odds operators). We assume that the spread betting commission is applied to the winning bets (as it is typical in online spread operators), whereas commission in parimutuel betting applies to the total amount (as in the Spanish regulation).

We show that, from the online bettor's point of view, it is preferable a GPT scheme, in the following sense: In equilibrium, the odds are not affected by the taxation under GPT; whereas a GBD scheme would reduce the odds and hence the bettors' utility. As opposed, in the parimutuel market GPT and GBD provide the same effect.

These results agree with the ones presented by Smith (2000) and Paton et al. (2002, 2001), who analyze the effect of the different taxation schemes in Australian, UK and USA betting markets. These results, however, are more focused on offline betting operators and government revenue. Moreover, they take into account the marginal cost of each bet. As opposed, we assume that these marginal costs are

[3] In case the odds summed up less than 100%, it would be possible, by betting an appropriate amount of money on each possible match outcome, to win a positive amount irrespectively of the final match outcome.

[4] The sum of the odds, called *overround*, provides a way to measure the operator advantage.

negligible.

There are other works that concentrate on parimutuel markets: Ottaviani and Sørensen (2009 and references herein) provide a model that explains the empirical evidence of underdogs overbet. These authors argue that this bias may be due to privately informed bettors. As opposed, we prove (Corollary **??**) that the spread bet operator would get a higher profit if the underdog wins the game. An immediate intuition suggests that bettors have incentives to bet for the favorite in spread markets. Hence, the existence of arbitrators betting in different online operators would provide an alternative explanation for the underdog bias in parimutuel markets[5].

Other works concentrate on fixed-odd markets. For example, Bag and Saha (2011 and references herein) study the externalities due to bribery in sports; and Levitt (2004) argues that the operators may achieve higher profits by an accurate prediction of the match outcome.

As far as we know, no similar research has been addressed for spread markets.

2. THE MODEL

The study the effect of taxation in the online market. Assume the regulator announces a tax, that could be a percentage υ on volume (GBD) or a percentage ρ on gross profit (GPT). The non-cooperative game has two steps:

Step 1: Bookmakers observe υ or ρ and announce their odds (fixed-odds) or commission (spread/parimutuel).

Step 2: bettors observe odds and commission on their respective bookmakers and choose to participate or not.

Fixed-odds bookmakers: Bookmaker chooses odds π_H and π_A. Bettors observe the odds and (simultaneously) choose $s_i \in \{D, H, A\}$.

If $s_i = D$, bettor i declines to bet (abstains) and its final payoff is zero.

If $s_i = H$, bettor i bets for the home team at odd π_H and its final payoff is

$$u_i = x_i \left(\frac{1}{\pi_H} - 1 \right) + (1 - x_i)(-1).$$

If $s_i = A$, bettor i bets for the away team at odd π_A and its final payoff is

$$u_i = x_i (-1) + (1 - x_i) \left(\frac{1}{\pi_A} - 1 \right).$$

Let $B_H = \{i \in B : s_i = H\}$ and $B_A = \{i \in B : s_i = A\}$. The bookmaker's final payoff is

$$\begin{aligned} u_k &= (1-\rho) \min \left\{ \begin{array}{c} (1-\upsilon)(\|B_H\| + \|B_A\|) - \frac{1}{\pi_H} \|B_H\|, \\ (1-\upsilon)(\|B_H\| + \|B_A\|) - \frac{1}{\pi_A} \|B_A\| \end{array} \right\} \\ &= (1-\rho) \left\{ (1-\upsilon)(\|B_H\| + \|B_A\|) - \max \left\{ \frac{1}{\pi_H} \|B_H\|, \frac{1}{\pi_A} \|B_A\| \right\} \right\}. \end{aligned}$$

[5] Work in progress.

Theorem 1 *Given υ and ρ, there exists a (strong) subgame perfect equilibrium. Moreover, the odds in equilibrium are characterized by the maximization problem*

$$\max (1-\rho)\left\{(1-\upsilon)\left[\int_{\pi_H}^{1} f(t)\,dt+\int_{0}^{1-\pi_A} f(t)\,dt\right]-\frac{1}{\pi_H}\int_{\pi_H}^{1} f(t)\,dt\right\}$$

subject to

$$\begin{aligned}\frac{1}{\pi_H}\int_{\pi_H}^{1} f(t)\,dt &= \frac{1}{\pi_A}\int_{0}^{1-\pi_A} f(t)\,dt \qquad (1)\\ \pi_H,\pi_A &\in [0,1],\pi_H+\pi_A\geq 1\end{aligned}$$

Corollary 2 *Assume there exists a unique (strong) subgame perfect equilibrium. Then,*

a) Taxes on volume (υ) decreases the bookmakers utility, increases the over-round and decreases the volume. Bettors utility is decreased.

b) Taxes on profit (ρ) decreases the bookmakers utility, but maintains the over-round and the volume. Bettors utility remains unchanged.

Spread bookmakers: Bookmaker chooses commission $c\in(0,1)$. Bettors observe the commission and (simultaneously) choose $s_i\in\{D\}\cup\{H,A\}\times[0,1]$.

If $s_i=D$, bettor i declines to bet and its final utility is zero.

If $s_i=(H,\pi_H)$, bettor i declares that it wants to bet for the home team at odd at most π_H.

If $s_i=(A,\pi_A)$, bettor i declares that it wants to bet for the away team at odd at most π_A.

The bookmaker matches (H,π_H)-bettors with (A,π_A)-bettors that satisfy $\pi_H\geq 1-\pi_A$ with odds π, $1-\pi$ such that: $\pi\leq\pi_H$ and $1-\pi\leq\pi_A$. The matching is done so that each π volume (H,π_H)-bettors is matched with a $1-\pi$ volume of (A,π_A)-bettors. The reason is that, in case home team wins, a $1-\pi$ volume of money is transferred from (A,π_A)-bettors to (H,π_H)-bettors, so that each (H,π_H)-bettor receives a gross winning (profit + bet):

$$\frac{1-\pi}{\pi}+1=\frac{1}{\pi}\geq\frac{1}{\pi_H}$$

hence granting their request to bet for the home team at odd at least π_H.

Analogously, in case away team wins, a π volume of money is transferred from (H,π_H)-bettors to (A,π_A)-bettors, so that each (A,π_A)-bettor receives a gross winning (profit + bet):

$$\frac{\pi}{1-\pi}+1=\frac{1}{1-\pi}\geq\frac{1}{\pi_A}$$

hence granting their request to bet for the away team at odd at least π_A.

Hence, π is chosen so that $\|\Gamma_H\|+\left\|\overline{\Gamma_H}\right\|\geq\|\Gamma_A\|$ and $\|\Gamma_A\|+\left\|\overline{\Gamma_A}\right\|\geq\|\Gamma_H\|$, where $\Gamma_H=\{i\in B_k:s_i=(H,\pi_H),\pi_H>\pi\}$, $\overline{\Gamma_H}=\{i\in B_k:s_i=(H,\pi)\}$, $\Gamma_A=\{i\in B_k:s_i=(A,\pi_A),\pi_A>1-\pi\}$ and $\overline{\Gamma_A}=\{i\in B_k:s_i=(A,\pi)\}$.

If $s_i=(H,\pi_H)$ with $\pi_H>\pi$, bettor i bets for the home team at odd π and its final payoff is

$$u_i=x_i(1-c)\left(\frac{1}{\pi}-1\right)+(1-x_i)(-1). \qquad (2)$$

If $s_i = (A, \pi_A)$ with $\pi_A > 1 - \pi$, bettor i bets for the away team at odd $1 - \pi$ and its final payoff is

$$u_i = x_i (-1) + (1 - x_i)(1 - c)\left(\frac{1}{1-\pi} - 1\right). \tag{3}$$

If $s_i = D$, or $s_i = (H, \pi_H)$ with $\pi_H < \pi$, or $s_i = (A, \pi_A)$ with $\pi_A < 1 - \pi$, bettor i does not bet and its final payoff is zero.

When $s_{i2} = \pi$, we have two cases:

Case 1: $\frac{\left\|\overline{\Gamma_H}\right\|}{\pi} \leq \frac{\left\|\overline{\Gamma_A}\right\|}{1-\pi}$. If $s_i = (H, \pi)$, then bettor i bets for the home team and his final payoff is (2). If $s_i = (A, \pi)$, then bettor i bets for the away team with probability $\frac{1-\pi}{\pi}\frac{\left\|\overline{\Gamma_H}\right\|}{\left\|\overline{\Gamma_A}\right\|}$ and its final payoff is

$$u_i = \frac{1-\pi}{\pi}\frac{\left\|\overline{\Gamma_H}\right\|}{\left\|\overline{\Gamma_A}\right\|}\left[x_i (-1) + (1 - x_i)(1 - c)\left(\frac{1}{1-\pi} - 1\right)\right].$$

Case 2: $\frac{\left\|\overline{\Gamma_H}\right\|}{\pi} \geq \frac{\left\|\overline{\Gamma_A}\right\|}{1-\pi}$. If $s_i = (A, \pi)$, then bettor i bets for the away team and his final payoff is (3). If $s_i = (H, \pi)$, then bettor i bets for the home team with probability $\frac{\pi}{1-\pi}\frac{\left\|\overline{\Gamma_A}\right\|}{\left\|\overline{\Gamma_H}\right\|}$ and its final payoff is

$$u_i = \frac{\pi}{1-\pi}\frac{\left\|\overline{\Gamma_A}\right\|}{\left\|\overline{\Gamma_H}\right\|}\left[x_i (1 - c)\left(\frac{1}{\pi} - 1\right) + (1 - x_i)(-1)\right].$$

Let $\Gamma = \Gamma_H \cup \overline{\Gamma_H} \cup \Gamma_A \cup \overline{\Gamma_A}$. The final payoff for the bookmaker is

$$\begin{aligned} u_k &= (1-\rho)\left[\min\left\{\begin{array}{c} c\left(\frac{1}{\pi} - 1\right)\left(\|\Gamma_H\| + \min\left\{1, \frac{\pi}{1-\pi}\frac{\left|\overline{\Gamma_A}\right|}{\left|\overline{\Gamma_H}\right|}\right\}\left\|\overline{\Gamma_H}\right\|\right) \\ c\left(\frac{1}{1-\pi} - 1\right)\left(\|\Gamma_A\| + \min\left\{1, \frac{1-\pi}{\pi}\frac{\left|\overline{\Gamma_H}\right|}{\left|\overline{\Gamma_A}\right|}\right\}\left\|\overline{\Gamma_A}\right\|\right) \end{array}\right\} - \upsilon\|\Gamma\|\right] \\ &= (1-\rho)\, c \min\left\{\begin{array}{c} \frac{1-\pi}{\pi}\left(\|\Gamma_H\| + \min\left\{\left\|\overline{\Gamma_H}\right\|, \frac{\pi}{1-\pi}\left\|\overline{\Gamma_A}\right\|\right\} - \upsilon\|\Gamma\|\right) \\ \frac{\pi}{1-\pi}\left(\|\Gamma_A\| + \min\left\{\left\|\overline{\Gamma_A}\right\|, \frac{1-\pi}{\pi}\left\|\overline{\Gamma_H}\right\|\right\} - \upsilon\|\Gamma\|\right) \end{array}\right\} \end{aligned}$$

Theorem 3 *Given υ and ρ, there exists a (strong) subgame perfect equilibrium with undominated strategies*[6]*. Moreover, the commission and odds in equilibrium are characterized by the maximization problem*

$$\max_{c\in(0,1)} (1-\rho)\, c \left[\begin{array}{c} \frac{1}{\pi}\int_{\frac{\pi}{1-c(1-\pi)}}^{1} f(t)\,dt - \max\left\{\int_{\frac{\pi}{1-c(1-\pi)}}^{1} f(t)\,dt, \int_{0}^{\frac{\pi(1-c)}{1-c\pi}} f(t)\,dt\right\} \\ -\upsilon\left(\int_{\frac{\pi}{1-c(1-\pi)}}^{1} f(t)\,dt + \int_{0}^{\frac{\pi(1-c)}{1-c\pi}} f(t)\,dt\right) \end{array}\right]$$

[6] Undominated strategies are required in order to avoid meaningless equilibria of the form "everybody chooses D". This requirement is not needed for strong subgame perfect equilibrium though.

subject to

$$\begin{aligned} \frac{1}{\pi}\int_{\frac{\pi}{1-c(1-\pi)}}^{1} f(t)\,dt &= \frac{1}{1-\pi}\int_{0}^{\frac{\pi(1-c)}{1-c\pi}} f(t)\,dt \qquad (4) \\ \pi, c &\in [0,1] \end{aligned}$$

Corollary 4 *Assume there exists a unique (strong) subgame perfect equilibrium. Then,*

a) Taxes on volume (υ) decreases the bookmakers utility, increases the commission and decreases the volume. Bettors utility is decreased.

b) Taxes on profit (ρ) decreases the bookmakers utility, but maintains the commission and the volume. Bettors utility remains unchanged.

Parimutuel bookmakers (Summary): Taxes on volume (υ) and on profit (ρ) are equivalent; both decrease bookmaker's utility, but maintain the commission and the volume. Bettors utility remains unchanged. See Vidal-Puga (2011) for details.

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