Sharing the revenues from broadcasting sport events

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Introduction

A study of how much money various professional sports leagues generates shows that the NFL made \$13 billion in revenue last season. The Major League Baseball, came second with \$9.5 billion and the Premier League third with \$5.3 billion.

Four of the top five leagues in revenue are in North America. However, 14 of the top 20 are football leagues that are mostly based in Europe.

For most leagues, the sale of broadcasting and media rights is one of the biggest source of revenue.

Sharing these sizable revenues among participating teams is not a straightforward problem and sharing rules vary across the world.

Introduction

The following four football leagues share the revenues from TV broadcasting according with the following criteria

Criteria → Country	Egalitarian	League performance: Points, ...	Social performance: TV audience, ...
England	50%	25%	25%
Germany		100%	
Italy	40%	30%	30%
Spain (new)	50%	25%	25%

The Spanish former criteria was a bargaining procedure. The teams and the league talk until they reach a share of the revenues.

Season 2015-16 in Spain

	TV audience (millions)	Points
First Team	157.3	91
Last team	10.9	32
Ratio	14.4	2.8

In general there is a high positive correlation between the ranking of the teams in the two criteria.

Nevertheless, in the TV audience criteria the differences between the amounts received by each team is larger than in the Points criteria.

Introduction

For the firm with the broadcasting TV rights which is more important, points or audience?

We think that audience is much more important than points.

Nevertheless some leagues give the same importance to both issues.

An explanation is that the league wants teams with similar levels.
People prefer to watch a match with two similar teams than a match with two teams in wich one of them is much more better than the other.
Thus, if we distribute the revenues in a more equitable way, teams will have a more similar level, the total audience of the league will be larger, and then the total revenue will be also larger.

Introduction

In this paper we do not discuss what kind of criteria should be used to divide the revenue among the teams.

We discuss how to divide a revenue (all, 25%,..) taking into account only the audiences of the teams.

We study two rules from a theoretical point of view.

We apply both to the case of the Spanish football league and we compare it with the real sharing.

Examples and ideas

Assume that 200 people watch a footbal match between teams i and j. The revenue generated by this match is 200€ (1€ per person). Besides we know the reasons why this people have seen the match.
In practice we know the audience and the revenue, but not the reasons.

1. 20 because they like football.
2. 100 because they like to see team i. For instance, they are supporters of team i.
3. 30 because they like to see team j. For instance, they are supporters of team j.
4. 50 because they consider that the match between teams i and j is interesting.

How we divide the total revenue (200) among teams i and j?

Examples and ideas

Our answer:

1. The 20€ generated by the viewers are divided equally among both teams.
2. The 100€ generated by the viewers are assigned to team i.
3. The 30€ generated by the viewers are assigned to team j.
4. The 50€ generated by the viewers are divided equally among both teams.

Team i will receive $10 + 100 + 0 + 25 = 135$.
Team j will receive $10 + 0 + 30 + 25 = 65$.

Examples and ideas

We argue that, in general, one might become a viewer of a game involving teams i and j for several reasons:

1. Because of being a fan of this sport per se. Thus, he/she views the game because is a football match.
2. Because of being a supporter of team i. Thus, he/she views the match because team i is playing.
3. Because of being a supporter of team j. Thus, he/she views the match because team j is playing.
4. Because of the match between i and j. Thus, he/she views the match because of the combination between teams i and j.

Examples and ideas

Problem: In practice we do not know why people see a match.

In practice we only know the total audience of a match.

In the rest of the paper, instead of dividing the revenue generated by the broadcasting TV rights, we divide the total audience of the matches.

The revenue will be divided between the teams in the same proportion than the total audience.

Examples and ideas

Example 1. Consider a league between 3 teams with the following audiences

$$A = \begin{pmatrix} 0 & 1200 & 1030 \\ 1200 & 0 & 230 \\ 1030 & 230 & 0 \end{pmatrix}$$

Several explanations of this data in terms of points 1 to 4 are possible. For instance,

First. All people belongs to group 4. No team has supporters.
This explanation is valid for any data.
Team 1 receives

$$\frac{1200}{2} + \frac{1030}{2} + \frac{1200}{2} + \frac{1030}{2} = 2230.$$

Analogously, we have that the sharing rule is

$$\begin{pmatrix} 2230 & 1430 & 1260 \end{pmatrix}$$

Examples and ideas

$$A = \begin{pmatrix} 0 & 1200 & 1030 \\ 1200 & 0 & 230 \\ 1030 & 230 & 0 \end{pmatrix}$$

Second. Team 1 has 1000 supporters, team 2 has 200 supporters and team 3 has 30 supporters. There is no people in groups 1 and 4.

This explanation is not valid for any data.

Team 1 receives 4000 (it plays 4 matches and in each match it has 1000 supporters)

Analogously we have that the sharing rule is

$$\begin{pmatrix} 4000 & 800 & 120 \end{pmatrix}$$

Examples and ideas

$$A = \begin{pmatrix} 0 & 1200 & 1030 \\ 1200 & 0 & 230 \\ 1030 & 230 & 0 \end{pmatrix}$$

Third. Team 1 has 800 supporters, team 2 has 100 supporters and team 3 has 30 supporters. There is 90 people in group 1. The rest of the people belongs to group 4. Thus,

	Group 1	Sup. 1	Sup. 2	Sup. 3	Group 4
1200	90	800	100		210
1030	90	800		30	110
230	90		100	30	10

Thus, the sharing rule is

$$\begin{pmatrix} 3700 & 800 & 420 \end{pmatrix}$$

Examples and ideas

Given some data, several explanations of groups 1 to 4 are possible, Thus, what to do?
In this paper we consider two scenarios:

First. No supporters. We assume that all people belongs to group 4.

This is the most pessimistic scenario for the computation of supporters.

We consider a rule in such scenario using game theory and claims problems.

We study properties of such rule and we give axiomatic characterizations.

Second. Mainly supporters. We take as much supporters as we can compatible with the data.

This is the most optimistic scenario for the computation of supporters.

We consider a rule in such scenario using optimization.

We study properties of such rule and we give axiomatic characterizations.

Examples and ideas

Example 1. The two scenarios correspond with.

- First:

$$\begin{pmatrix} 2230 & 1430 & 1260 \end{pmatrix}$$

- Second:

$$\begin{pmatrix} 4000 & 800 & 120 \end{pmatrix}$$

As a conclusion of our results we give two bounds for computing rules. Namely, the allocation to each team should be included in the interval given by both scenarios.

In Example 1 team 1 should receive something between 2230 and 4000, team 2 between 800 and 1430 and team 3 between 120 and 1260.

The model

For each pair of teams $i, j \in N$, we denote by a_{ij} the broadcasting audience (number of viewers) for the game played by i and j at i's stadium.

We use the notational convention that $a_{ii} = 0$, for each $i \in N$.

Let $A = (a_{ij})_{(i,j) \in N \times N}$ denote the resulting matrix with the broadcasting audiences generated in the whole tournament played by teams in N.

Let $||A|| = \sum_{i,j \in N} a_{ij}$ the total audience of the tournament.

The model

A (broadcasting sports) **problem** is a duplet (N, A), where

$N \in \mathcal{N}$ is the set of **teams**.

$A = (a_{ij})_{(i,j)\in N\times N} \in \mathcal{A}_{n\times n}$ is the **audience matrix**.

For each $(N, A) \in \mathcal{P}$, and each $i \in N$, let $\alpha_i(A)$ denote the total audience achieved by team i, i.e.,

$$\alpha_i(A) = \sum_{j\in N}(a_{ij} + a_{ji}).$$

Notice that

$$||A|| = \frac{\sum_{i\in N} \alpha_i(A)}{2}$$

The model

A (sharing) **rule** is a mapping that associates with each problem an allocation indicating the amount each team gets from the total revenue generated by broadcasting games.

Without loss of generality, we normalize the revenue generated by each viewer to 1 (to be interpreted as the "pay per view" fee).

Thus, formally, $R : \mathcal{P} \rightarrow \mathbb{R}^n$ is such that, for each $(N, A) \in \mathcal{P}$,

$$\sum_{i \in N} R_i (N, A) = ||A||.$$

Example 1. Let (N, A) be such that $N = \{1, 2, 3\}$ and

$$A = \begin{pmatrix} 0 & 1200 & 1030 \\ 1200 & 0 & 230 \\ 1030 & 230 & 0 \end{pmatrix}$$

Then

$$||A|| = 4920$$

$$\alpha(A) = (4460, 2860, 2520).$$

First scenario: No supporters

First, we consider the rule associated to the first scenario. There is no supporters and then the revenues from each game are divided equally among the two playing teams.

Equivalently, given our normalization convention, each team is awarded half of its total audience.Formally,

Equal Split, **ES**: For each $(N, A) \in \mathcal{P}$, and each $i \in N$,

$$ES_i (N, A) = \frac{\alpha_i}{2}.$$

Example 1.

$$A = \begin{pmatrix} 0 & 1200 & 1030 \\ 1200 & 0 & 230 \\ 1030 & 230 & 0 \end{pmatrix}$$

In this case

$$S = \begin{pmatrix} 2230 & 1430 & 1260 \end{pmatrix}$$

First scenario: No supporters

A **TU game**, is a pair (N, v), where N is the set of agents and $v : 2^N \rightarrow \mathbb{R}$ satisfies that $v(\varnothing) = 0$.

We say that (N, v) is **convex** if, for each pair $S, T \subset N$ and $i \in N$ such that $S \subset T$ and $i \notin T$,

$$v(T \cup \{i\}) - v(T) \geq v(S \cup \{i\}) - v(S).$$

The **core** is defined as

$$C(N, v) = \left\{ x \in \mathbb{R}^N : \sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \geq v(S) \ \forall\ S \subset N \right\}.$$

The **Shapley value** (Shapley, 1953) is defined as

$$Sh_i(N, v) = \frac{1}{n!} \sum_{\pi \in \Pi_N} \left[v(Pre(i, \pi) \cup \{i\}) - v(Pre(i, \pi)) \right],$$

First scenario: No supporters

We associate with each (broadcasting sports) problem $(N, A) \in \mathcal{P}$ a TU game (N, v_A) where, for each $S \subset N$, $v_A(S)$ denotes the total audience of the games played by the teams in S. Namely,

$$v_A(S) = \sum_{\substack{i,j \in S \\ i \neq j}} a_{ij} = \sum_{\substack{i,j \in S \\ i < j}} (a_{ij} + a_{ji}).$$

Notice that, for each problem $(N, A) \in \mathcal{P}$ and each $i \in N$, $v_A(\{i\}) = 0$.

First scenario: No supporters

Example 1.

$$A = \begin{pmatrix} 0 & 1200 & 1030 \\ 1200 & 0 & 230 \\ 1030 & 230 & 0 \end{pmatrix}$$

$$\begin{array}{ccccc} S & \{1,2\} & \{1,3\} & \{2,3\} & \{1,2,3\} \\ v_A(S) & 2400 & 2060 & 460 & 4920 \end{array}$$

and

$$Sh(N, v_A) = (2230, 1430, 1260) = ES(N, A).$$

First scenario: No supporters

Our main findings regarding the game v_A are

Theorem

(a) The equal split rule coincides with the Shapley value for TU-games described above. Formally, for each problem (N, A) we have that

$$Sh(N, v_A) = ES(N, A).$$

(b) The game v_A is convex. Thus the core is non-empty and the Shapley value belongs to the core.

First scenario: No supporters

The core is easily characterized.
For each pair of teams, we divide the audience of the games played by both teams in any way among them.
Each team receives the aggregation of these amounts, across the rest of the teams. Formally :

Theorem

$x = (x_i)_{i \in N} \in C(N, v_A)$ if and only if, for each $i \in N$, there exist $\left(x_i^j\right)_{j \in N \setminus \{i\}}$ satisfying three conditions:

(i) $x_i^j \geq 0$, for each $j \in N \setminus \{i\}$;

(ii) $\sum_{j \in N \setminus \{i\}} x_i^j = x_i$, for each $i \in N$;

(iii) $x_i^j + x_j^i = a_{ij} + a_{ji}$, for each pair $i, j \in N$, with $i < j$.

First scenario: No supporters

A *claims problem* is a triple (N, c, E) consisting of a population $N \in \mathcal{N}$, a claims profile $c \in \mathbb{R}^n_+$, and an *endowment* $E \in \mathbb{R}_+$ such that $\sum\limits_{i \in N} c_i \geq E$.

Let $C \equiv \sum_{i \in N} c_i$.

A *rule* associates with each problem $(N, c, E) \in \mathcal{D}$ an allocation $R(N, c, E) \in \mathbb{R}^n$ fsatisfying the following two conditions:

(i) for each $i \in N$, $0 \leq R_i(N, c, E) \leq c_i$ and

(ii) $\sum\limits_{i \in N} R_i(N, c, E) = E$.

First scenario: No supporters

The *proportional* rule, P, yields awards proportionally to claims, i.e., for each $(N, c, E) \in \mathcal{D}$, $P(N, c, E) = \frac{E}{C} \cdot c$.

The *constrained equal-awards* rule, CEA, selects, for each $(N, c, E) \in \mathcal{D}$, the vector $(\min\{c_i, \lambda\})_{i \in N}$, where $\lambda > 0$ is chosen so that $\sum_{i \in N} \min\{c_i, \lambda\} = E$.

The *constrained equal-losses* rule, CEL, selects, for each $(N, c, E) \in \mathcal{D}$, the vector $(\max\{0, c_i - \lambda\})_{i \in N}$, where $\lambda > 0$ is chosen so that $\sum_{i \in N} \max\{0, c_i - \lambda\} = E$.

The *Talmud* rule is a hybrid between the above two. More precisely, for each $(N, c, E) \in \mathcal{D}$, it selects

$$T(N, c, E) = \begin{cases} CEA(N, \frac{1}{2}c, E) & \text{if } E \leq \frac{1}{2}C \\ \frac{1}{2}c + CEL(N, \frac{1}{2}c, E - \frac{1}{2}C) & \text{if } E \geq \frac{1}{2}C \end{cases}$$

First scenario: No supporters

We associate with each (broadcasting sports) problem (N, A) a claims problem $\left(N, c^A, E^A\right)$ where

$c_i^A = \alpha_i(A)$, for each $i \in N$, and

$E^A = ||A||$.

First scenario: No supporters

In **Example 1**

$$A = \begin{pmatrix} 0 & 1200 & 1030 \\ 1200 & 0 & 230 \\ 1030 & 230 & 0 \end{pmatrix}$$

we have that $E = 4920$ and

i	1	2	3
c_i	4460	2860	2520
$P = T$	2230	1430	1260
CEA	1640	1640	1640
CEL	2820	1220	880

Our main findings are:

Theorem

(a) *The proportional rule coincides with the Talmud rule and with the equal split rule. Hence the proportional rule belongs to the core.*
(b) *The CEA rule could be outside the core.*
(c) *The CEL rule could be outside the core.*

Second scenario: Mainly supporters

We consider the opposite scenario. We assume that nobody belongs to group 4, i.e., nobody is a joint fan of both teams in a game.

In other words, we assume that when somebody decides to watch a game, it is because he/she is a fan of one of the teams or because he/she is a fan of the specific sport being considered.

In this scenario, we believe each team should receive the revenues generated by its fans, whereas the revenue coming from the generic sport fans should be divided equally among all teams.

Second scenario: Mainly supporters

Let b_0 denote the number of generic sport fans. For each $i \in N$, let b_i denote the number of fans of team i.

Thus, for each pair $i, j \in N$, with $i \neq j$.

$$a_{ij} = b_0 + b_i + b_j + \varepsilon_{ij}.$$

Viewers from group 4 are therefore collected in $\{\varepsilon_{ij}\}_{i,j \in N}$. As in this scenario we are assuming that nobody belongs to group 4, our aim is to make $\{\varepsilon_{ij}\}_{i,j \in N}$ as small as we can (given the audience data).

Thus, we take $b = \{b_i\}_{i=0}^{n} \in \mathbb{R}^{n+1}$ such that

$$\min_{b \in \mathbb{R}^{n+1}} \sum_{i,j \in N, i \neq j} \varepsilon_{ij}^2 = \min_{b \in \mathbb{R}^{n+1}} \sum_{i,j \in N, i \neq j} \left(a_{ij} - b_0 - b_i - b_j\right)^2. \tag{1}$$

Second scenario: Mainly supporters

Let $\hat{b}_0$ and $\left\{\hat{b}_i\right\}_{i\in N}$ denote the solutions to (1). Besides, for each pair $i,j \in N$, $i \neq j$ we denote

$$\hat{\varepsilon}_{ij} = a_{ij} - \widehat{b_0} - \widehat{b_i} - \widehat{b_j}$$

We now divide the audience according to the following principles:

$(P1)$ $\hat{b}_0$ is divided equally among all teams.
$(P2)$ $\hat{b}_i$ $\left(\widehat{b_j}\right)$ is assigned to team i (j).
$(P3)$ $\hat{\varepsilon}_{ij}$ is divided equally between teams i and j.

Applying those principles we can define a rule $R^b(N, A)$ where, for each problem $(N, A) \in \mathcal{P}$ and each $i \in N$, the allocation for team i is

$$R_i^b(N, A) = (n-1)\,\widehat{b_0} + 2(n-1)\,\widehat{b_i} + \sum_{j \in N \setminus \{i\}} \frac{\widehat{\varepsilon_{ij}} + \widehat{\varepsilon_{ji}}}{2}. \qquad (2)$$

Second scenario: Mainly supporters

Unfortunately, the minimization problem (1) cannot be solved.

We then remove one of the teams $k \in N$, and consider the following minimization problem. We take $b = \left\{ b_0, \{b_i\}_{i \in N \setminus \{k\}} \right\} \in \mathbb{R}^n$ such that

$$\min_{b \in \mathbb{R}^n} \sum_{i,j \in N, i \neq j} \varepsilon_{ij}^2 \tag{3}$$

where

$$\varepsilon_{ij} = \begin{cases} a_{ij} - b_0 - b_i - b_j & \text{if } k \notin \{i,j\} \\ a_{ij} - b_0 - b_i & \text{if } k = j \\ a_{ij} - b_0 - b_j & \text{if } k = i \end{cases}$$

Then, we can apply principles $(P1)$, $(P2)$ and $(P3)$ to the solution of this problem in order to obtain a rule as in (2).

Second scenario: Mainly supporters

The potential issue that could arise is that the derived allocation would depend on k (the removed team).

The next theorem shows that this is not the case.

Theorem

For each (N, A) and each pair $i, k \in N$, let $R_i^{b,k}(N, A)$ be the allocation obtained by applying formula (2) to the minimization problem (3). Then,

$$R_i^{b,k}(N, A) = \frac{(n-1)\alpha_i - ||A||}{n-2}.$$

Second scenario: Mainly supporters

We define the **concede and divide** rule as

$$CD_i\left(N,A\right)=\frac{\left(n-1\right)\alpha_i-||A||}{n-2}.$$

Second scenario: Mainly supporters

We can express the rule in the following alternative way:

$$CD_i(N,A) = \alpha_i - \frac{\sum\limits_{j,k \in N \setminus \{i\}} (a_{jk} + a_{kj})}{n-2}.$$

Notice that

- α_i is the audience in the $2(n-1)$ games played by team i.
- $\frac{\sum\limits_{j,k \in N \setminus \{i\}} (a_{jk} + a_{kj})}{n-2}$ is the audience (the average per $n-1$ games) in the games played by the rest of the teams.

Thus, the rule is assigning to each team the differential audience with respect to the audience per $n-1$ games that the rest of the teams yield (in the remaining games they play).

Example 1.

$$A = \begin{pmatrix} 0 & 1200 & 1030 \\ 1200 & 0 & 230 \\ 1030 & 230 & 0 \end{pmatrix}$$

In this case

$$O = \begin{pmatrix} 4000 & 800 & 120 \end{pmatrix}$$

The axiomatic approach

The first axiom we consider says that if two teams have the same audiences, then they should receive the same amount.

Equal treatment of equals: For each $(N, A) \in \mathcal{P}$, and each pair $i, j \in N$ such that $a_{ik} = a_{jk}$, and $a_{ki} = a_{kj}$, for each $k \in N \setminus \{i, j\}$,

$$R_i(N, A) = R_j(N, A).$$

The second axiom says that revenues should be additive on A. Formally,

Additivity: For each pair (N, A) and $(N, A') \in \mathcal{P}$

$$R\left(N, A + A'\right) = R\left(N, A\right) + R\left(N, A'\right).$$

The axiomatic approach

The third axiom says that if a team has no audience, then such a team gets no revenue.

Null team: For each $(N, A) \in \mathcal{P}$, and each $i \in N$, such that $a_{ij} = 0 = a_{ji}$, for each $j \in N$,

$$R_i(N, A) = 0.$$

Alternatively, the next axiom says that if a team nullifies the audience of all the games it plays (for instance, due to some kind of boycott), then the allocation of such a team should decrease exactly by the total audience of such a team. Formally,

Nullifying team: For each $(N, A), (N, A') \in \mathcal{P}$ such that there exists $k \in N$ (the nullifying team) satisfying $a'_{ij} = a_{ij}$ when $k \notin \{i, j\}$ and $a'_{ij} = 0$ when $k \in \{i, j\}$ we have that

$$R_k(N, A') = R_k(N, A) - \alpha_k(A).$$

The next axiom says that each team should receive at least 0.

Non negativity. For each $(N, A) \in \mathcal{P}$ and $i \in N$,

$$R_i(N, A) \geq 0.$$

The next axiom says that the rule should belong to the core of (N, v_A). Namely,

Core selection: For each $(N, A) \in \mathcal{P}$

$$R(N, A) \in C(N, v_A).$$

The axiomatic approach

The next axiom says that each team should receive, at most, the total audience of the games played by the team.

Maximum aspirations: For each $(N, A) \in \mathcal{P}$ and each $i \in N$

$$R_i(N, A) \leq \alpha_i(A).$$

The next axiom says that if the audience of team i is, game by game, not smaller than the audience of team j, then team i could not receive less than team j.

Monotonicity: For each $(N, A) \in \mathcal{P}$ and each pair $i, j \in N$, such that, for each $k \in N \setminus \{i, j\}$, $a_{ik} \geq a_{jk}$ and $a_{ki} \geq a_{kj}$ we have that

$$R_i(N, A) \geq R_j(N, A).$$

The axiomatic approach

The next axiom refers to the incremental effect of adding a single additional viewer to a game. It states that the additional revenue should be shared equally among the involved teams in such a game. Formally,

Equal sharing of additional viewers: For each pair $(N, A), (N, \hat{A}) \in \mathcal{P}$ such that $a_{ij} = \hat{a}_{ij} = 0$, for each pair $(i, j) \neq (i_0, j_0)$, and $a_{i_0, j_0} + 1 = \hat{a}_{i_0, j_0}$,

$$R_{i_0}(N, \hat{A}) - R_{i_0}(N, A) = R_{j_0}(N, \hat{A}) - R_{j_0}(N, A).$$

The axiomatic approach

Properties	equal split	concede and divide
Equal treatment of equals	Yes	Yes
Additivity	Yes	Yes
Null team	Yes	No
Nullifying team	No	Yes
Non negativity	Yes	No
Core Selection	Yes	No
Maximum aspirations	Yes	Yes
Monotonicity	Yes	Yes
Equal sharing of additional viewers	Yes	Yes

The axiomatic approach

Theorem

The equal split rule is the unique rule satisfying

(a) Equal treatment of equals, additivity and null team.

(b) Equal sharing of additional viewers, additivity and null team.

Theorem

The concede and divide rule is the unique rule satisfying

Equal treatment of equals and nullifying team.

An empirical application

We now apply the results of our paper to the Spanish Football League.

The data have been obtained from some web pages specialized in sports.

La Liga does not provide data of the audiences.

Conclusions

We have presented a stylized model to deal with the problem of sharing the revenues from broadcasting sports events using the audiences data.

We have provided normative foundations for two rules, which have distinguishing merits.

- The concede and divide rule reflects the (potentially different) fan base of each team.
- The equal split rule is supported by game theory and bankruptcy problems.

We have also provided as a case study an empirical application for the Spanish Football League.